AD 606041

University of Oklahoma Research Institute

CRITERIA FOR CASUALTY PRODUCTION
and
PRELIMINARY FLAME PATTERN ANALYSIS

by

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Contract DA 18-035 AMC-116(A)

TM-1454-1-1

September 15, 1964

Contract No. DA 18-035-AMC-116(A)

OURI Project No. 1454

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ABSTRACT

Some of the results contained in this paper have been obtained as a by-product of other research, but have been collected in this report to establish a frame of reference part of which is applicable to flame pattern configuration. The first part of the paper is directed towards investigating the distribution of casualty producing material from the dynamics of dispersion. Some of this information may prove useful in establishing theoretical upper bounds on the region of Lethality.

The last part of the paper is concerned with an analytical model for flame distributions resulting from a bomb dispersed gel, and of a flamethrower as based on empirical data. This data will serve as a lower limit of desired effects since it is currently obtainable. A number of curves are included showing the area of the target covered as a function of the orientation of the strike relative to the target center.

CRITERIA FOR CASUALTY PRODUCTION

1. Statement of Problem:

There are several facets of casualty production that must be investigated if a sensible evaluation of criteria is to be reached. The following discussion will be directed towards the problem of casualty production as a consequence of non-nuclear weapons.

The first facet of the problem is to establish a criterion of casualty production through the determination of parameters necessary for a particle to break the surface of the skin either by impact and penetration, or by burning This problem will involve questions of size, shape, and possibly velocity of the particle as well as questions of protective shielding, be it clothing or some kind of armour.

Closely related to the foregoing is the associated problem of establishing criteria for casualty production as dependent upon the type of activity involved and the nature or location of the wound. Detailed investigation of this problem will confront the investigators with psychological characteristics of the personnel involved as related to the tactical situation as well as physiological criteria. Further consideration of this problem will not be considered here.

A third problem has to do with the distribution of the material effecting the casualty production about the point of impact, that is, the establishment of a Region of Lethality and the associated density

of casualty production within this region. This then relates to the second problem through a consideration of the attitude of the personnel.

The introductory comments of this discussion will be rather general, but is intended as a springboard for outlining more specific requirements without producing an extensive lacuna or gap in concepts.

2. Parameters of Casualty Production:

As a working definition, we shall consider an individual to be a casualty if he is unable to effectively use the equipment at hand due to physical injury that he has sustained. The most tangible form of injury that can be measured physiologically would be a lesion of some description. Lesions, in turn, can be produced by particles penetrating the skin through impact or by burning.

In constructing any model of casualty production a criterion of mathematical simplicity is desired. However, simplicity to permit ease of calculation must be constrained through the use of logical connections of the employed parameters.

Any criteria of casualty production should be stated in terms of a probability distribution that will increase monotonically in terms of of the critical parameters.

Ballistic wounding as defined by the piercing or breaking of the skin through impact will be dependent on dynamical parameters

(1) momentum, mv

(2) energy, $\frac{1}{2} mv^2$

(3) density of particles in the neighborhood of the personnel

By considering the parameters of mass, m, and velocity, v, in their

nascent states, illogical connections between parameters may arise which in no sense can be theoretically justified.

The following conditions can be expected to hold in any probability function for casualty production by "ballistic wounding":

(1) If the momentum and energy are too small, there will be no casualty production implying a threshold value below which the probability of wounding is zero. In other words, there is a lower bound of momentum and energy below which a casualty cannot be produced.

(2) As momentum and energy increase, the probability of wounding will increase to a maximum of one, provided the density of wounding particles remains sufficiently high.

(3) Since momentum $M = mv$ and energy $E = \frac{1}{2} mv^2$, any increase in one of the parameters implies an increase in the other. Hence, the probability should be monotone increasing with m and v.

(4) As the distance from the center of impact increases, the density of wounding particles decreases and hence the probability of wounding decreases.

In considering lesions caused by "flame materials" a slightly different framework must be considered in that the dynamical feature of momentum and energy of motion are not of maximal importance, but density of flammable material is. For flame materials, it is required that

(1) size of "flame" supporting particle is large enough to burn for sufficient time at lesion producing temperatures.

(2) density of "flame" supporting material be sufficient so that fire prevention and control techniques are reduced in effectiveness, preferably to a point of being completely ineffective.

3. Probability Function Forms for Casualty Production:

From the foregoing discussion of parameters it becomes evident that the probability function of "ballistic wounding" and "flame lesions" as criteria for casualty production are fundamentally different.

Since "ballistic wounding" depends upon the dynamical parameters of momentum and energy, it is conjectured that a probability density dependent only on them should assume the form of

$$P = 1 - e^{a(mv - mv_o) + 2b(\frac{1}{2} mv^2 - \frac{1}{2} mv_o^2)}$$

or more simply

$$P = 1 - e^{m\left[a(v - v_o) + b(v - v_o^2)\right]}$$

Where a is the coefficient of momentum change and $2b$ is the coefficient of energy change corresponding to a particle of a given mass m. The velocity v_o is a threshold velocity, below which no wounding takes place for that size of particle. If sufficient data is available, the coefficients a and b can be estimated from a least squares criterion applied to the equivalent linear equation

$$\ln(1 - P) = am\Delta v + bm\Delta v^2$$

where $\Delta v = v - v_o, \Delta v^2 = v^2 - v_o^2$

If the dependence on distance from the center of impact is to be incorporated in the probability statement, it will have to be multiplied by some factor near Gaussian in form such as $\exp(-r^2)$. The true nature of this factor will depend upon the dynamical paths of the particles.

The probability function for causing "flame lesions" is fundamentally a time and density dependent function and hence $P = P(d, t)$. For example it has been determined that 20 gms/m^2 of white phosphorus are needed to produce casualties.

4. Region of Lethality

The parameters of concern for the production of casualties are different for ballistic wounding and wounding caused by flame lesions. The determination of the region of lethality resulting from the burst of a conventional H.E. warhead can be approximated from certain theoretical concepts. In order to obtain a solution in closed form, however, certain s mplifying assumptions need to be made. Some of these same assumptions are plausable for determining the distribution of fuel from a flame projectile. Possible modeling structures for flame warheads could have a controlled H.E. burst to distribute the flame fuel in a prescribed pattern from an aerial burst or a projectile. A hybrid projectile would also have the goal of the H. E. fracturing or otherwise disrupting the target surface enabling the fuel to more effectively ignite the target material.

It is assumed that the particles of the burst are finite in number (N), have constant mass (m), and that their density or dispersion will reach a lower limit of effectiveness. It is furthermore

assumed that the particle distribution will be governed by gravitational attraction and air resistance. The altitude of the burst will be considered, but assumed to be near enough to ground level so that the acceleration of gravity will be effectively constant.

The differential equation for the motion of a typical particle in vector form is

$$\ddot{R} = F(\dot{R}) + gE_1,$$

where superposed dots denote time derivatives and capital letters denote vectors. In particular R is the position vector, E_1 is a unit vector directed downward, g the acceleration of gravity assumed to be constant and F a vector function of the velocity. The true nature of the vector F is not known explicitly, however its behavior lies somewhere between a linear response to velocity and a quadratic response to velocity.

5. Air Resistance Proportional to the Velocity:

If the vector function F is proportional to the velocity the equation of motion becomes

$$\ddot{R} = -k\dot{R} + gE_1$$

This has the first integral of

$$\dot{R} = -\left[(gE_1 - kV_o)e^{-kt} - gE_1\right]/k$$

where V_o is the initial velocity of the particular particle or globule when the warhead explodes. This equation yields the second integral of

$$R = \left[(gE_1 - kV_o)(e^{-kt} - 1) + k(gt - hk)E_1\right]/k^2$$

where h is the height above the ground of the burst. That is at $t = 0$ $R \cdot E_1 = -h$.

5a. Region of Lethality for Ballistic Wounding:

To inflict damage to the target by ballistic wounding, it is assumed that the magnitude of velocity of a particle must exceed a given velocity. In other words, if $|\dot{R}| < v_L$, there will be no effective damage to the target. The lethal region is bounded by the envelope of the set of points for which $|\dot{R}| = v_L$, provided of course that the particle density is sufficiently high.

Each particle in the burst will define a separate trajectory which will be determined by its own initial condition, i.e., the vector velocity of the warhead and the direction and velocity that the particle takes relative to the warhead. The geometry and distribution of material on the warhead will affect the region through density considerations.

On the boundary of the Lethal region $\dot{R} = V_L$, from which it follows at the boundary that

$$V_L + kR_B = V_o + (gt_B - hk)\, E_1 \tag{1}$$

where t_B is the time for the particular particle to reach the boundary and R_B is the position vector denoting the boundary. It also follows that

$$V_L = -\frac{gE - kV_o}{k}\, e^{-kt_B} + \frac{gE_1}{k} \tag{2}$$

Eliminating V_L from equations (1) and (2) gives a vector equation relating R_B and t_B

$$kR_B = (V_o - \frac{gE_1}{k})(1 - e^{-kt_B}) + (gt_B - hk)\,E_1 \tag{3}$$

The initial velocity V_o, consists of two parts

i) the velocity contributed by the projectile

$$V_p = v_p\,(E_1 \sin \omega + E_2 \cos \omega) \tag{4}$$

ii) and the initial velocity imparted to the individual particle on exploding the warhead

$$V_e = v_e\,(E_1 \cos \alpha + E_2 \cos \beta + E_3 \cos \gamma) \tag{5}$$

where E_1 is a unit vector oriented downward, E_2 a unit vector in the horizontal direction of the direction of motion of the projectile and E_3 a unit vector perpendicular to E_1 and E_2, $(E_1 \times E_2 = E_3)$.

Let the vector equation of the boundary be

$$R_B = aE_1 + rE_2 + dE_3 \tag{6}$$

where a is the altitude, r the range and d the lateral displacement of the particle from the point of explosion. Equation (3) and (6) now gives

$$ka = (v_p \sin \omega + v_e \cos \alpha - \frac{g}{k})\,(1 - e^{-kt_B}) + gt_B - hk$$

$$kr = (v_p \cos \omega + v_e \cos \beta)\,(1 - e^{-kt_B}) \tag{7}$$

$$kd = v_e \cos \gamma\,(1 - e^{-kt_B})$$

The proportionality constant k can be eliminated from two of these equations as well as the exponential term to yield

$$rv_e \cos\gamma = d(v_p \cos\omega + v_e \cos\beta) \tag{8}$$

Another relation independent of the exponential term is

$$(ka - gt_B + hk)\ v_e \cos\gamma = d(v_p \sin\omega + v_e \cos\alpha - \frac{g}{k}) \tag{9}$$

Solving this equation for t_B yields

$$t_B = \frac{k(a+h)}{g} + \frac{d}{kv_e \cos\gamma} - \frac{d(v_p \sin\omega + v_e \cos\alpha)}{gv_e \cos\gamma} \tag{10}$$

$$= \frac{k(a+h)}{g} + \frac{d(\frac{1}{r} - (v_p \sin\omega + v_e \cos\alpha)/_g)}{v_e \cos\gamma}$$

$$= \frac{k(a+h)}{g} + \frac{r\left[\frac{1}{k} - (v_p \sin\omega + v_e \cos\alpha)/_g\right]}{v_p \cos\omega + v_e \cos\beta}$$

$$= \frac{k(a+h)}{g} + \frac{r\left[g - k(v_p \sin\omega + v_e \cos\alpha)\right]}{gk\,(v_p \cos\omega + v_e \cos\beta)}$$

As a first approximation $1 - e^{-kt_B} = kt_B$ if $kt_B < 1$,

from this it follows that

$$kd = v_e \cos\gamma\,(1 - e^{-kt_B}) \approx kv_e \cos\gamma t_B$$

hence

$$kd = kv_e \cos\gamma \left[\frac{k(a+h)}{g} + \frac{d}{kv_e \cos\gamma} - \frac{d(v_p \sin\omega + v_e \cos\alpha)}{gv_e \cos\gamma}\right] \tag{11}$$

It now follows that

$$k^2(a+h)\ v_e \cos\gamma = kd(v_p \sin\omega + v_e \cos\alpha) \tag{12}$$

Using equation (8) and (12) in the identity

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \tag{13}$$

we get

$$dv_e \cos \alpha = k(a + h)v_e \cos \gamma - dv_p \sin \omega$$

$$dv_e \cos \beta = rv_e \cos \gamma - dv_p \cos \omega$$

Hence

$$d^2v_e^{\,2}(1 - \cos^2\gamma) = \left[k^2(a + h)^2 + r^2\right] v_e^{\,2}\cos^2\gamma - 2dv_ev_p\cos\gamma \cdot \left[k(a + h) \sin \omega + r \cos \omega\right] + d^2v_p^{\,2}$$

This can be rewritten as

$$v_e^{\,2}\cos^2\gamma \left[k^2(a + h)^2 + r^2 + d^2\right] - 2dv_ev_p\cos\gamma \left[k(a + h) \sin \omega + r \cos \omega\right] + d^2 \left[v_p^{\,2} - v_e^{\,2}\right] = 0$$

or

$$(a + h,\ r,\ d)\begin{pmatrix} k^2v_e^{\,2}\cos^2\gamma & 0 & -v_ev_p \cos \gamma \sin \omega \\ 0 & v_e \cos^2 \gamma & -v_ev_p \cos \gamma \sin \omega \\ -v_ev_p \cos \gamma \sin \omega & -v_ev_p \cos \gamma \cos \omega & v_e^{\,2}\cos^2\gamma + (v_p^{\,2} - v_e^{\,2}) \end{pmatrix}\begin{pmatrix} a+h \\ r \\ d \end{pmatrix} = 0$$

This is a homogeneous quadratic form and thus represents a degenerate conic. However, an assignment to the initial attitude of the projectile (assignment of ω) and a specification of γ which determine points on a cone with its axis horizontally sideways, constraint on the boundary values of a, r, and d are then given. By further assigning a value to d compatible with these constraints a level line mapping is obtainable.

Taking d as a parameter, the equation can be rewritten as

$$\frac{\left[(a+h) - \dfrac{dv_p \sin\omega}{kv_e \cos\gamma}\right]^2}{\dfrac{d^2 v_e^{\,2} \sin^2\gamma}{k^2 v_e^{\,2} \cos^2\gamma}} + \frac{\left[r - \dfrac{dv_p \cos\omega}{v_e \cos\gamma}\right]^2}{\dfrac{d^2 v_e^{\,2} \sin^2\gamma}{v_e^{\,2} \cos^2\gamma}}$$

which is an ellipse with its center at

$$(a+h,\ r) = \left(\frac{dv_p \sin\omega}{kv_e \cos\gamma},\ \frac{dv_p \cos\omega}{v_e \cos\gamma}\right)$$

and semi-axes of lengths

$$d \tan\gamma \quad \text{and} \quad d \tan\gamma/k$$

in the r and $a + h$ directions respectively.

From equation (12) find

$$v_e \cos\gamma = \frac{d(v_p \sin\omega + v_e \cos\alpha)}{k(a+h)}$$

which is then substituted into the equation for the degenerate conic

$$\left[r - \frac{k(a+h)\, v_p \cos\omega}{v_p \sin\omega + v_e \cos\alpha}\right]^2 + d^2$$

$$= \left\{\frac{v_e^{\,2}}{(v_p \sin\omega + v_e \cos\alpha)^2} - \left[1 - \frac{v_p \sin\omega}{v_p \sin\omega + v_e \cos\alpha}\right]^2\right\} k^2 (a+h)^2$$

$$= \left\{\frac{v_e^{\,2}}{(v_p \sin\omega + v_e \cos\alpha)^2} - \frac{v_e^{\,2} \cos^2\alpha}{(v_p \sin\omega + v_e \cos\alpha)^2}\right\} k^2 (a+h)^2$$

$$= \frac{k^2 v_e^{\,2} \sin^2\alpha\, (a+h)^2}{(v_p \sin\omega + v_e \cos\alpha)^2}$$

An interpretation of this equation is that any particle emitted from the projectile along the right circular cone α (axis vertical) will have its boundary for the region of lethality on a circle that has been translated forward from the point of explosion. For a given h and ω, the maximum translation occurs with $\alpha = \pi/2$. The translation is then

$$k(a + h) \cos \omega \qquad \omega \neq 0$$

and the radius of the boundary circle is

$$\frac{kv_e (a + h)}{v_p \sin \omega} \qquad \omega \neq 0$$

The ratio of the translation to the radius of dispersion is given by $v_p \cos \omega / v_e \sin \alpha$. In order that the region of lethality be essentially circular it is necessary that this ratio be less than 1 for some α. In other words it is sufficient if $v_p \cos \omega < v_e$ to have a circular region of lethality.

5b. Region of Lethality for Flame Lesions:

In order to produce a casualty by flames the velocity of the globules of burning material is not of primary concern. Instead we are concerned with the geometrical distribution of the material.

Again assuming that the air resistance is proportional to the velocity, the foregoing equations for R, $\dot{R}$, and $\ddot{R}$ still apply. We are now concerned with the when and where the globules of material strike the ground. The material will reach the ground when $R \cdot E_1 = 0$. Consider for simplicity the material leaving the warhead with a vector velocity

$$V_o = aE_1 + b(\cos \Theta E_2 \cdot \sin \Theta E_3).$$

Due to horizontal symmetry we can for simplicity assume that $\theta = 0$. The material will now reach the ground when

$$(g - ka)(e^{-kb} - 1)/k + (gt - hk) = 0$$

The distance from the center of explosion will be given by $R \cdot E_2$ evaluated at the time value obtained from the foregoing equation and is given by

$$R \cdot E_2 = -b(e^{-kt} - 1)/k = r$$

Note that the limiting value for r is b/k.

The time can be obtained from these equations in terms of a, b, h and r to get

$$t = \left[gr + k(bh - ar)\right] /bg.$$

Substituting this value of t into the equation for r, gives on rearranging terms

$$\left[e^{-\frac{kr}{b}}\right]^{1 - \frac{ak}{g}} = \left(1 - \frac{kr}{b}\right) e^{\frac{hk^2}{g}}$$

Since $r < b/k$ let $r = b/_{nk}$ for a number $n > 1$. Then a value of t can be obtained from

$$r = \frac{b}{k} \left(1 - e^{-kt}\right)$$

in particular $t = -\frac{1}{k} \operatorname{Ln} \left(1 - \frac{1}{n}\right) = \frac{1}{k} \operatorname{Ln} \left(\frac{n}{n - 1}\right)$.

Substituting the assumed value for r and the calculated t into the time equation

$$tbg = kbh + r(g - ak)$$

gives the value of h needed under the prescribed conditions, i.e.

$$h = \frac{g}{k^2}\left[\mathrm{Ln}\left(\frac{n}{n-1}\right) - \frac{1}{n}\right] + \frac{a}{nk}$$

In firing from a static platform $\underline{\underline{a = 0}}$ we obtain the following results

n	$\mathrm{Ln}\,\frac{n}{n-1} = kt$	$\mathrm{Ln}\,\frac{n}{n-1} - \frac{1}{n}$	k^2h
2	0.69315	$\mathrm{Ln}\,2 - \frac{1}{2} = 0.19315$	6.18080 ft/sec^2
3	0.40547	$\mathrm{Ln}\,\frac{3}{2} - \frac{1}{3} = 0.07213$	2.30816 ft/sec^2
5	0.22314	$\mathrm{Ln}\,\frac{5}{4} - \frac{1}{5} =$ 0.02[illegible]4	0.74048 ft/sec^2

If suitable values of k are known approximately satisfying the assumptions, the height of the burst required for distributing the flames over prescribed circles can be approximated.

6. <u>Air Resistance Proportional to the Square of the Velocity</u>:

Under the assumption of air resistance being proportional to the velocity squared, the differential equation can be written as

$$\ddot{R} + kv\dot{R} = gE_1$$

or

$$\ddot{R} + kT\,\dot{R}\cdot\dot{R} = gE_1$$

Where T is the unit target multiplying by $\dot{R}\cdot$ gives

$$\dot{R}\cdot\ddot{R} + k\dot{R}\cdot T\,\dot{R}\cdot\dot{R} = \frac{d}{dt}\left(\frac{\dot{R}\cdot\dot{R}}{2}\right) + k\dot{R}\cdot T\,\dot{R}\cdot\dot{R} = g\dot{R}\cdot E$$

which in turn can be rewritten as

$$\frac{d}{dt}\left[\frac{1}{2}\dot{R}\cdot\dot{R}e^{-2k\int\dot{R}\cdot Tdt}\right] = g\,\dot{R}\cdot E\,e^{-2k\int\dot{R}\cdot Tdt}$$

But $$\int\dot{R}\cdot Tdt = \int vdt = \int\frac{ds}{dt}dt = \int ds = s$$

hence the differential equation of motion becomes

$$\frac{d}{dt}\left[\frac{1}{2}v^2e^{-2ks} = g\,\dot{R}\cdot Ee^{-2ks}\right]$$

This has a first integral

$$\frac{1}{2}v^2 = e^{2ks}\left[c + g\int\dot{R}\cdot E\,e^{-2ks}\,dt\right]$$

The parameter k, according to John E. Younger (Advanced Dynamics - Ronald Press, 1958, p. 97), is of the order of magnitude

$$k \approx 10^{-3}/\text{ft}.$$

Some further approximations can be made towards getting the velocity explicitly. On rewriting and integrating by parts

$$\int\dot{R}\cdot E\,e^{-2ks}\,dt = \int e^{-2ks}\,E\cdot dR = \int e^{-2ks}\,E\cdot Tds$$

$$= \frac{E\cdot T}{2k}e^{-2ks} + \frac{1}{2k}\int\kappa e^{-2ks}\,E\cdot Nds$$

$$= -\frac{E\cdot T}{2k}e^{-2ks} - \frac{\kappa E\cdot N}{4k^2}e^{-2ks} + \frac{1}{4k^2}\int e^{-2ks}\,E\cdot\left[\kappa'N + \kappa N'\right]ds$$

where N is the unit normal vector and primes denote differentiation with respect to arc length.

If the motion is planar $N' = -\kappa T$ $(N' = \tau B - \kappa N)$

$$\int \dot{R}\cdot E e^{-2ks}\, dt = -\frac{-e^{-2ks}}{4k^2} E\cdot \left[2kT + \varkappa N\right] + \frac{1}{4k^2}\left\{\int \varkappa' e^{-2ks} E\cdot N ds - \int \varkappa^2 e^{-2ks} E\cdot T ds\right\}$$

Furthermore if the curvature $\varkappa$ is approximately constant

$$\frac{4k^2 + \varkappa^2}{4k^2} \int e^{-2ks} \dot{R}\cdot E\, dt \approx -\frac{e^{-2ks}}{4k^2} E\cdot \left[2kT + \varkappa N\right]$$

$$\therefore \frac{1}{2} v^2 \approx e^{2ks} \left[c - \frac{g e^{-2ks}}{4k^2 + \varkappa^2} E\cdot \left\{2kT + \varkappa N\right\}\right]$$

At $t = 0$, $s = 0$, $v = v_o$; T and N must be specified but these depend upon the dynamical paths of the material. Ignoring this we find

$$c = \frac{1}{2} v_o^2 + \frac{gE\cdot \left[2kT_o + \varkappa N_o\right]}{4k^2 + \varkappa^2}$$

$$v^2 \approx e^{2ks} \left\{v_o^2 + \frac{2gE \left[2kT_o + \varkappa N_o\right]}{4k^2 + \varkappa^2} - \frac{2gE\cdot \left[2kT + \varkappa N\right] e^{-2ks}}{4k^2 + \varkappa^2}\right.$$

$$v \approx e^{ks} \left\{v_o^2 + \frac{2g}{4k^2 + \varkappa^2} E\cdot \left[2k(T_o - Te^{-2ks}) + \varkappa(N_o - Ne^{-2ks})\right]\right\}^{\frac{1}{2}}$$

$$\approx \left\{\left(v_o e^{ks}\right)^2 + \frac{2g}{4k^2 + \varkappa^2} E\cdot \left[2k\left(T_o e^{2ks} - T\right) + \varkappa\left(N_o e^{2ks} - N\right)\right]\right\}^{\frac{1}{2}}$$

This particular equation of motion readily yields an expression for the velocity and hence could be used for obtaining some insight in "ballistic wounding" criteria. However, it does not yield a vector solution for the displacements in closed form and consequently the dimensions of the distribution as desired for flame investigations are unknown.

In order to use the foregoing approximations, explicit statements for R would be necessary in order to get appropriate values for the curvature κ.

7. Special Empirical Configurations:

Field studies have been carried out studying the effect of flames from flame throwers and Napalm bombs. The primary objective of one of these studies was to determine measures for defense against conventional flame weapons. A by-product was the establishment of flame patterns resulting from them. In both situations it was observed that

(1) A flame distribution divides into a central high heat zone and an outer splatter zone.

(2) Convective currents funnel the flame towards the center.

Relative dimensions of the high heat zone and splatter zone have been determined. In order to more completely assess the lethal area produced by these two conventional modes of flame delivery it becomes necessary to assess the relative areas of the two zones under a probability distribution. In a technical report for Weapons Systems Analysis Branch, USA CDC Artillery Agency, Fort Sill, Oklahoma, a nomograph was prepared for determining the common area between two overlapping circles distributed with a Gaussian Distribution about a given aim point. As a first order model it is conjectured that the flame distributions of flame thrower and Napalm bombs can be approximated by two eccentric circles. The smaller inner circle representing the high intensity zone.

Two separate models have been considered, based upon data from the report "Individual Defense against Napalm". In this report it was

observed that a 110 gal. Napalm bomb distributed in a high intensity zone nearly elliptical with dimensions of 10 by 65 meters. The boundary of the splatter zone was contiguous at a point of impact and had dimensions of 30 by 100 meters as in Figure 1.

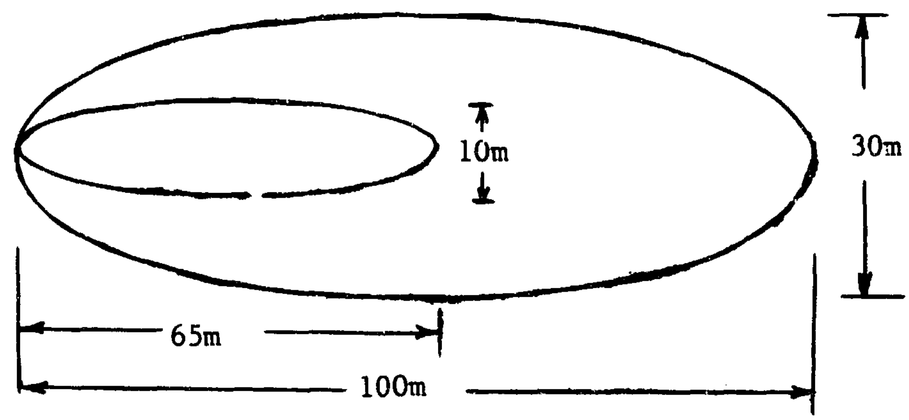


Figure 1

In the same report; it was observed that a flame thrower at 69 feet had a much more eccentric pattern in the high intensity zone of 4 ft. by 30 ft. surrounded by a splatter zone of 15 ft. by 90 ft. This time, however, the zones were more nearly concentric as in Figure 2.

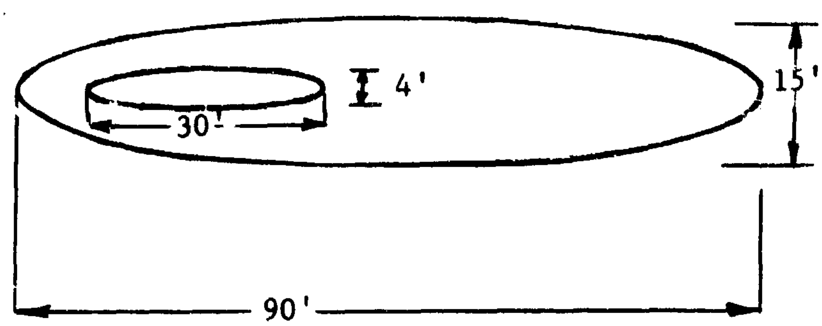


Figure 2

In order to model these configurations we make the following assumptions:

(1) The target under fire is circular.

(2) The flame pattern can be reasonably approximated by suitably oriented circles.

At this point it becomes necessary to specialize to the different patterns.

7a. Bomb Configuration:

In constructing this model it is again observed that

A. High intensity and splatter zone are contiguous on the leading edge.

B. The zone areas have the ratio

High Intensity: Splatter Zone = 1:5

from which it follows that if the high intensity zone has a radius of R, then the splatter zone has a radius of $\sqrt{5}$ R.

C. Normalize the configuration by assigning the target area to a radius of one (1).

D. The burst studied will be distributed with a circular Gaussian distribution about the target center for the high intensity zone, but with a constant direction of delivery. See Figure 3.

The area damage effected by the high intensity zone depends upon the distance from the center of the target. This data is partially documented in Table I and exhibited in Figure 5. The area of the splatter zone depends on the relative direction from the target center as well as the distance. Agains referring to Figure 3, one observes that the distance of the center of the circle representing the splatter zone is given by

$$1 - b = \sqrt{(1 - a)^2 - 2R(\sqrt{5} - 1)(1 - a)\cos\phi + R^2(\sqrt{5} - 1)^2}$$

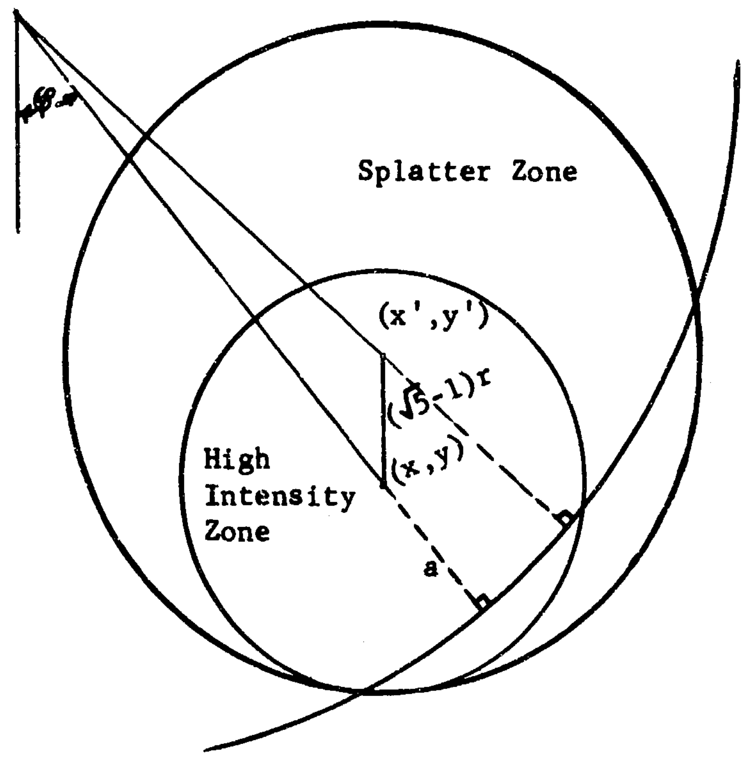


Figure 3

The distances (1-b) were obtained by interpretation from the data of Table II which is a listing of values obtained from

$$1 - b = \sqrt{(1-a)^2 - 2R(1 - a)\cos\varphi + R^2}$$

where R was assigned the values of 0.1, 0.3, and 0.5. The angle φ was assigned values at 18° increments ranging from 0° to 180° and a the distance from the target boundary was assigned increments of 0.1 ranging from zero the boundary to 1, the target center.

For a bomb type configuration the relative dimensions of the parts to Figure 3 are given by Table II.

The distance from the center of the target to the center of the splatter zone in terms of a, the distance from the target boundary, r,

the distance between centers of the high intensity zone and splatter zone and the relative direction from the line of fire to impact center is given for selected value of r in Table III.

Once the distance from the center of the target is obtained, the relative area of the target covered by the composite pattern can be readily obtained from Figure 5. Area coverage curves for this pattern are exhibited in Figures 6 - 10 inclusive.

In order to obtain the lethality of the composite pattern, the total area must be separated into two distinct parts. To the high intensity zone, a high probability would be assigned, but the splatter zone would have a small probability. It follows that the probability of casualty production by such a pattern would then be

$$p = p_h A_h + p_s A_s$$

where p_h = probabiltiy of a casualty in the high intensity zone

p_s = probability of a casualty in the splatter zone

A_h = area of high intensity zone within the target

A_s = area of splatter zone within the target.

It is conjectured that from this data a modeling pattern can be derived for investigating the composite effects of several palm type bombs.

7b. <u>Flame Thrower Configurations</u>:

The configurations resulting from the use of flame throwers depend to some extent upon the manner of employment. Since the target is observable, except for the obscuring resulting from the flame and ensuing smoke, the effect can be monitored by the operator. In this section we

will not go into much detail since it is not felt to be as necessary.

The primary observed difference is that the high intensity zone is no longer contiguous to the splatter zone boundary, but does have its boundary at the center of the splatter zone.

A second variation lies in the observed ratio of areas, i.e.,

High Intensity : Splatter Zone = 1 : 11

Utilizing the same simplifying assumptions, the configuration is typified by Figure 4. The relative dimensions in Figure 4 are given in Table III.

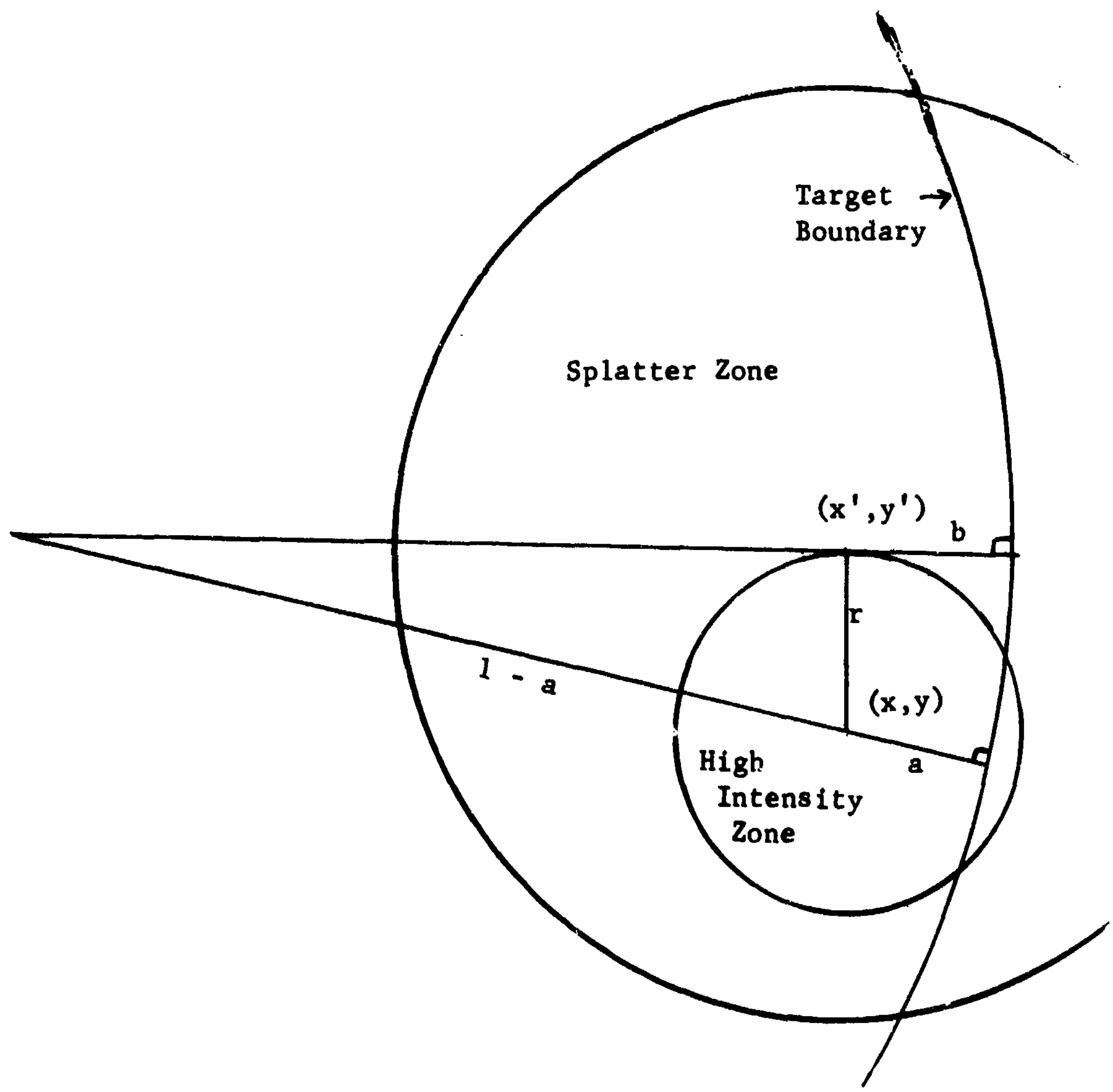


Figure 4

Curves exhibiting the composite areas of coverage are given in Figures 11 to 15 inclusive.

8. <u>Summary and Recommendations</u>:

The Lethality patterns as described in some detail give some indication of the probability of casualty production for a single burst, be it a bomb or a flame thrower. These configurations will be used as a guide in developing pattern effects and attempting to prescribe ways of optimizing these effects.

The data of this analysis will be subsequently related to the distribution function resulting from the delivery mode and expected values for casualty production will be estimated.

Subsequently, flame patterns will be designed to give reasonable approximations to the composite pattern of a single burst. This will be analyzed on the OURI SADI Mark IV to determine effects resulting from a number of bursts deployed against a target. The SADI Mark IV will in particular be used to determine statistical limits of effectiveness that can be anticipated to hold when delivery errors of different types are hypothesized.

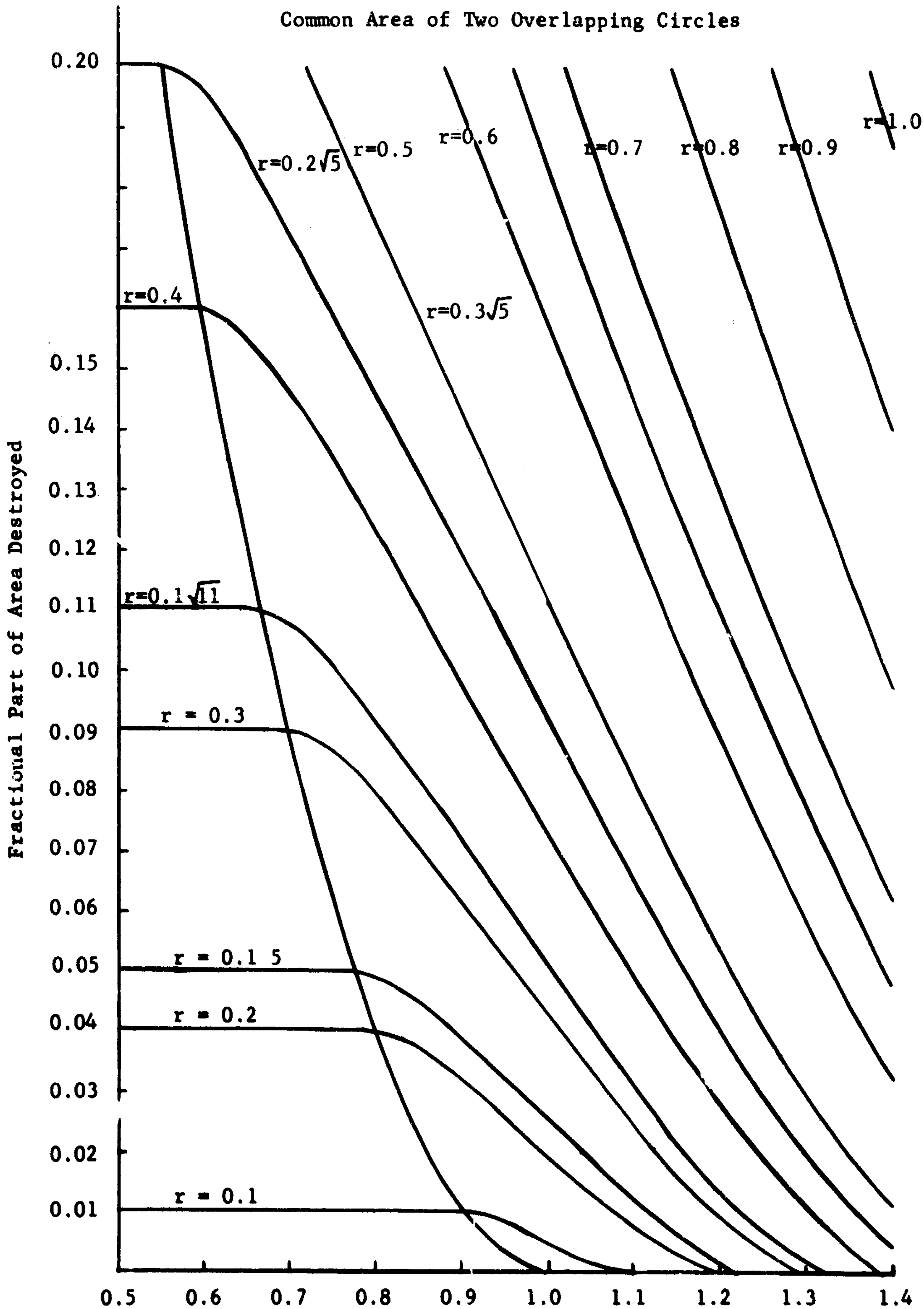


Figure 5 - Distance Between Centers of Circles

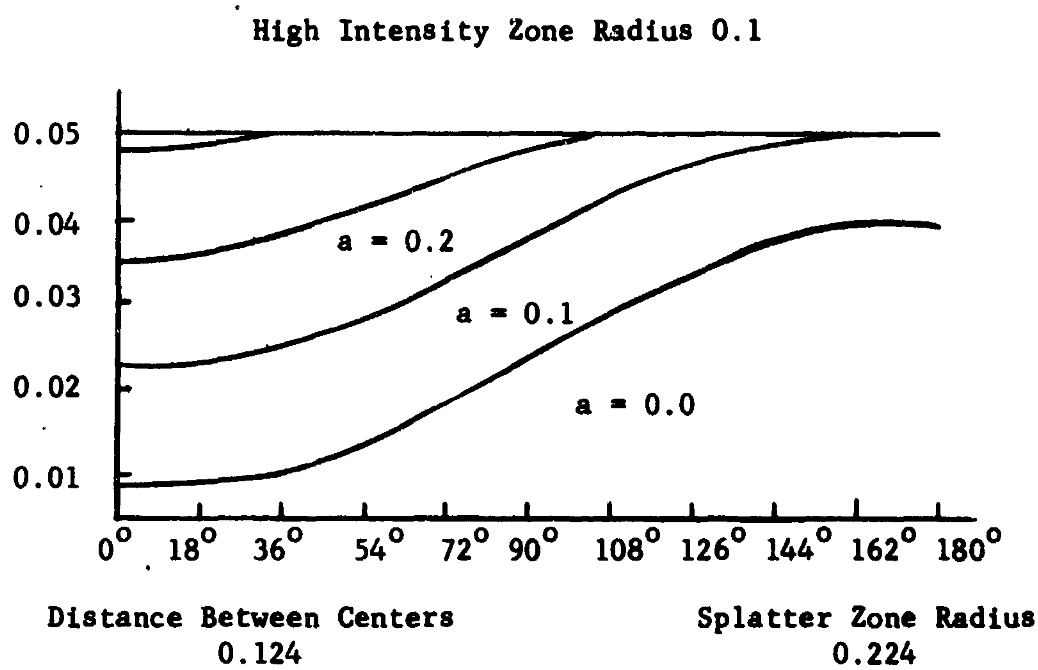


Figure 6

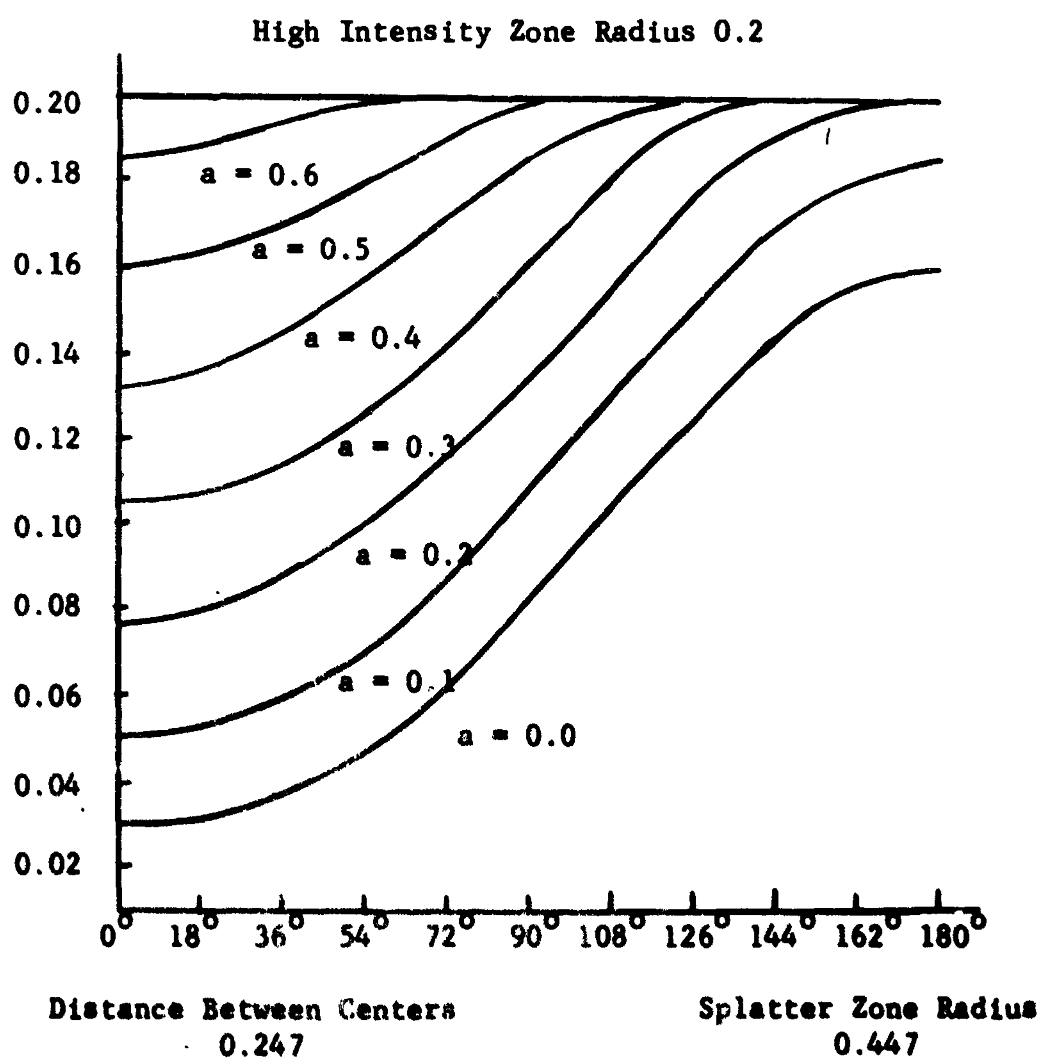
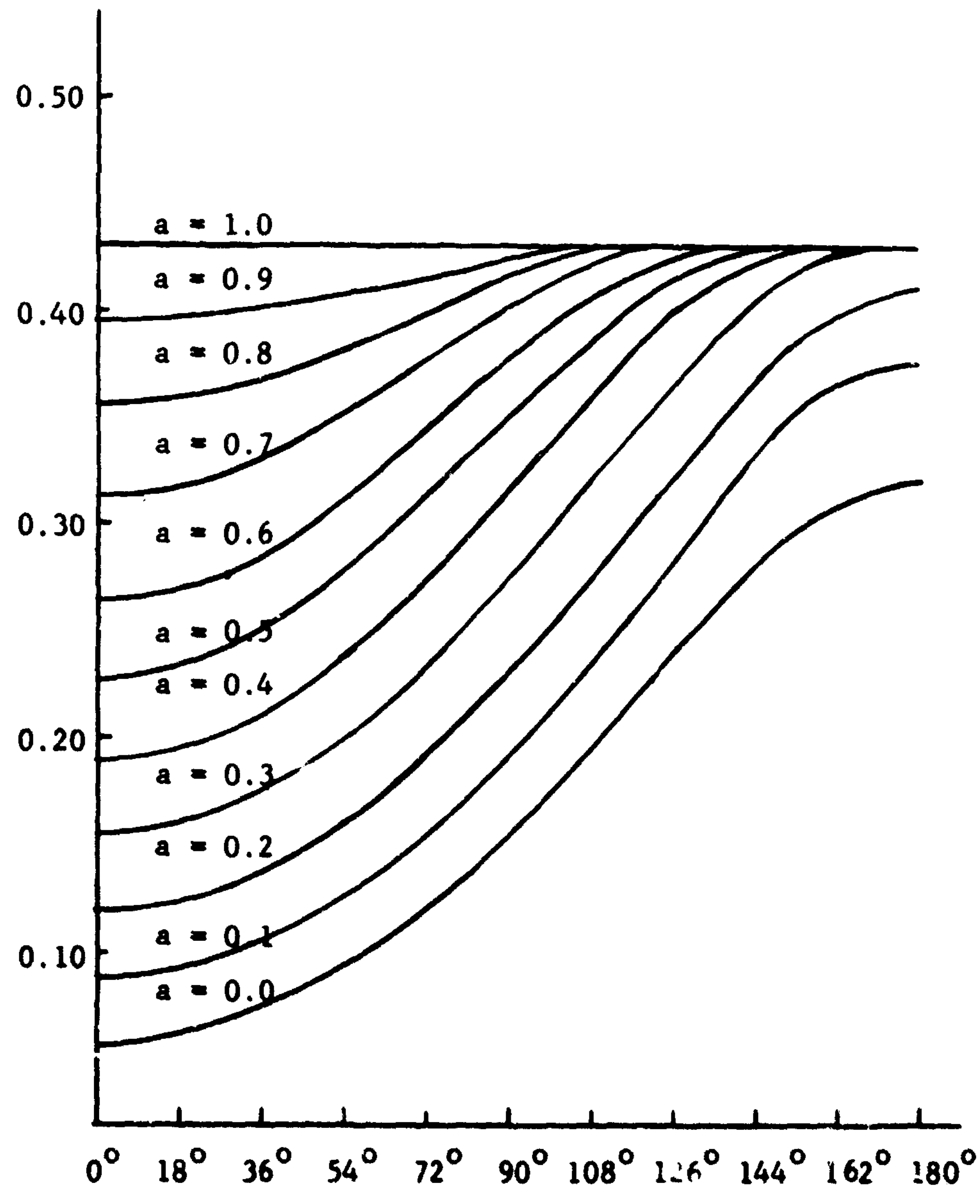


Figure 7

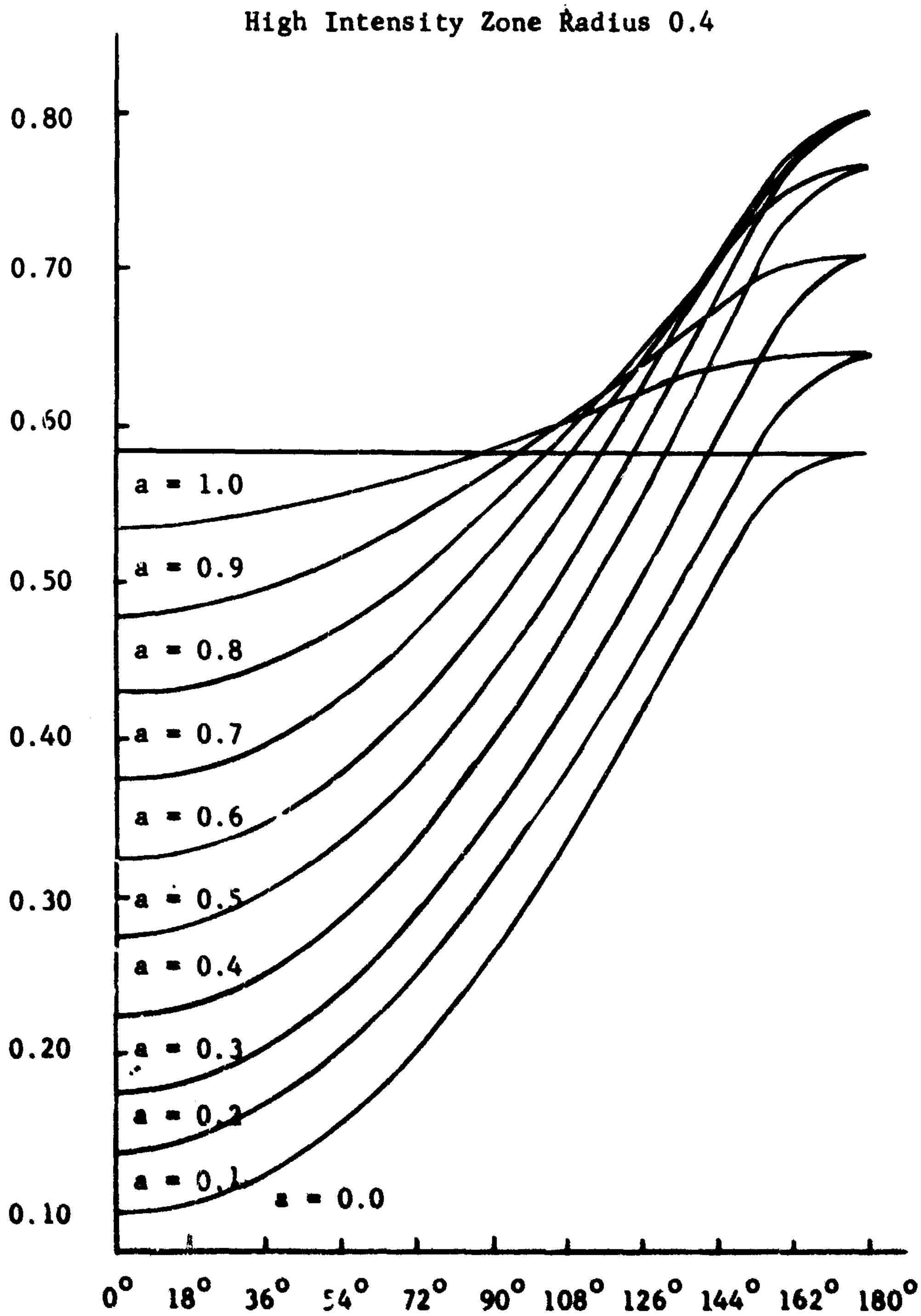
High Intensity Zone Radius 0.3



Distance Between Centers 0.371

Splatter Zone Radius 0.671

Figure 8

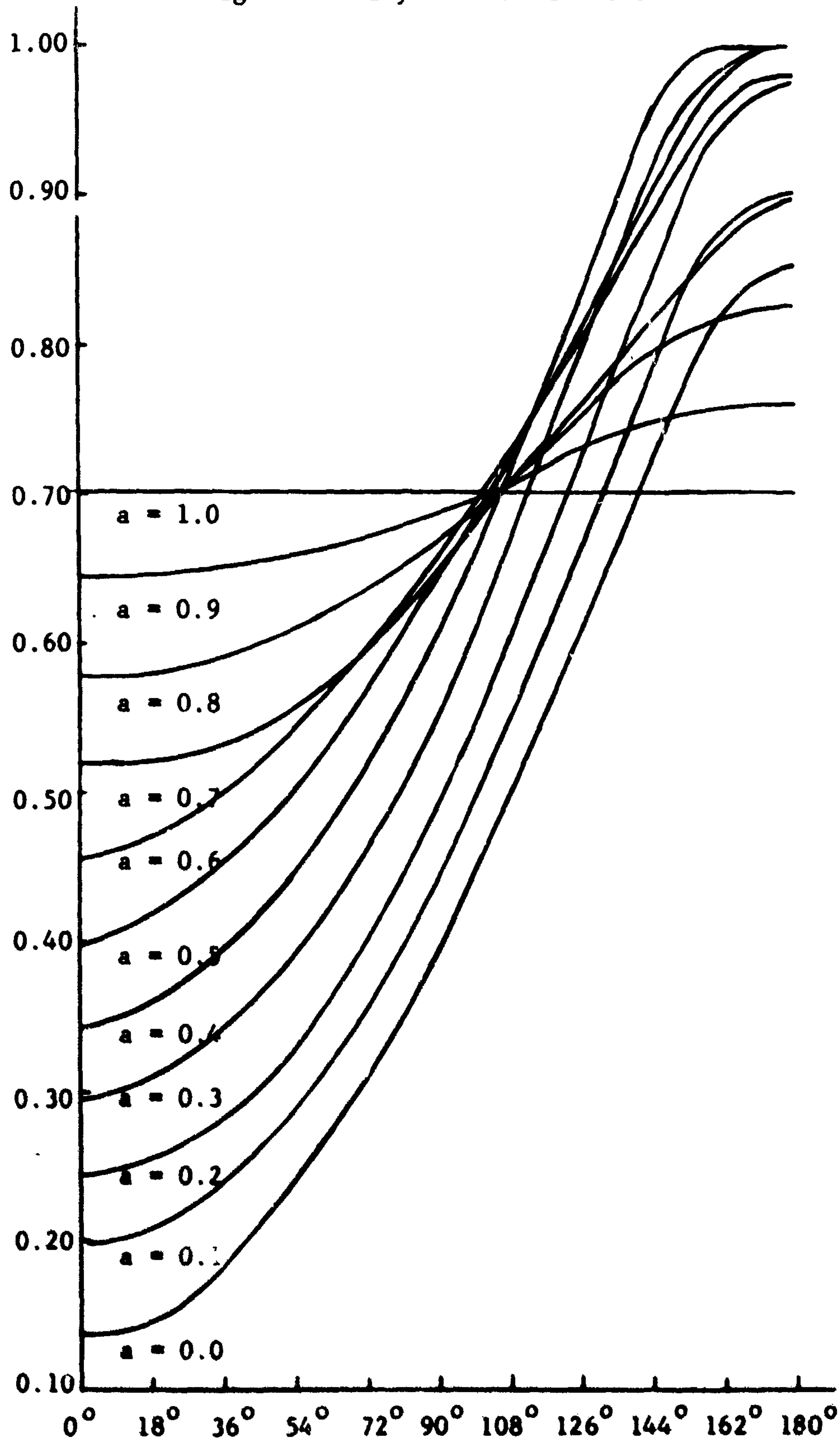


Distance Between Centers 0.494

Splatter Zone Radius 0.894

Figure 9

High Intensity Zone Radius 0.5

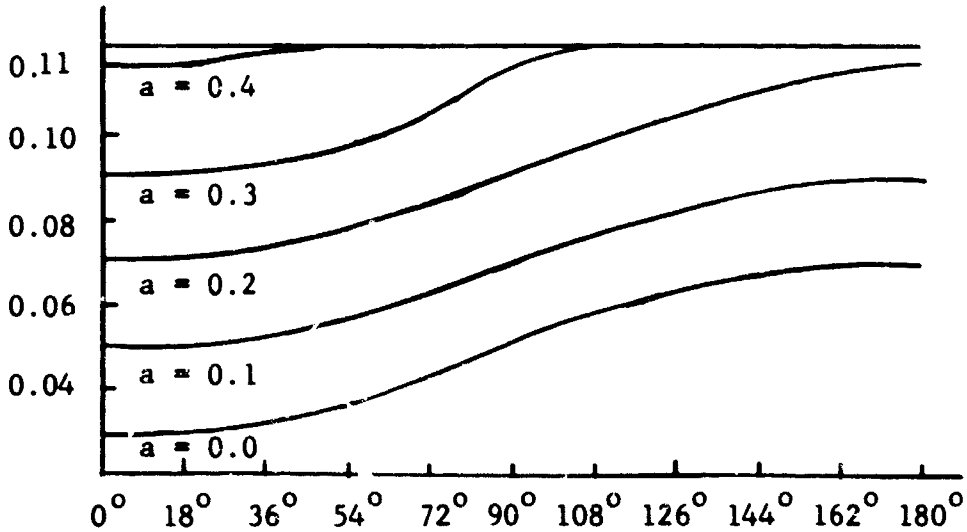


Distance Between Centers
0.618

Splatter Zone Radius
1.118

Figure 10

High Intensity Zone Radius 0.1

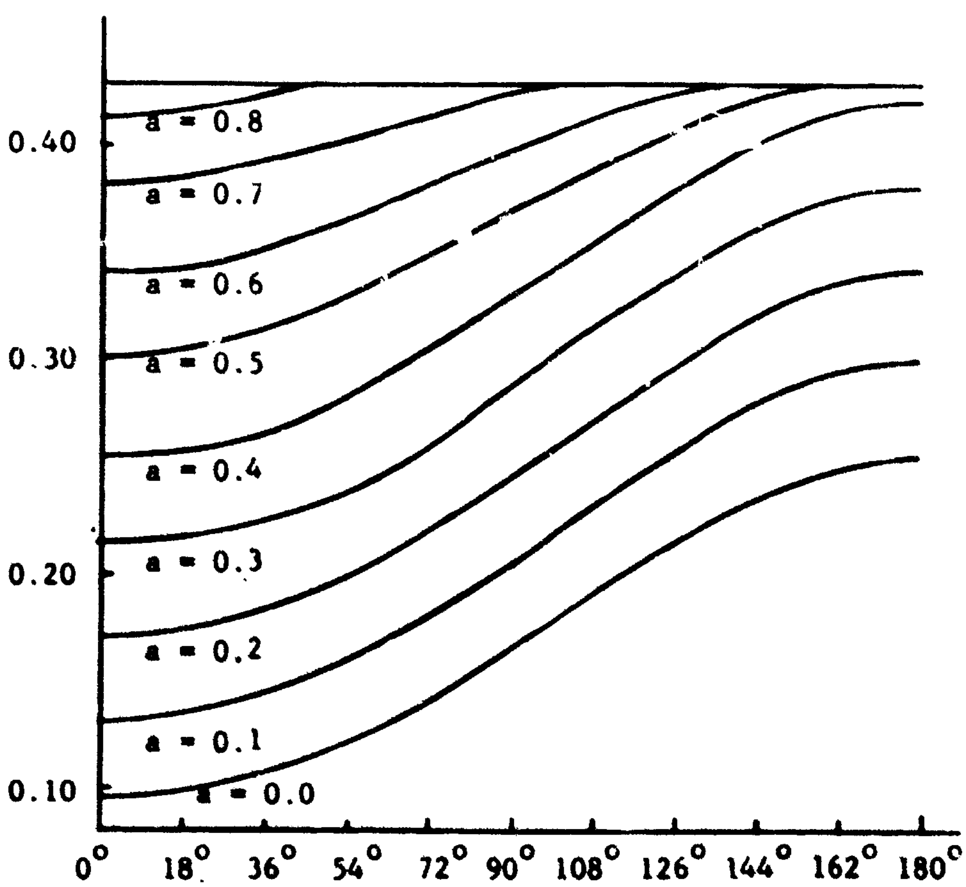


Distance Between Centers 0.1

Splatter Zone Radius 0.332

Figure 11

High Intensity Zone Radius 0.2



Distance Between Centers 0.2

Splatter Zone Radius 0.663

Figure 12

High Intensity Zone Radius 0.3

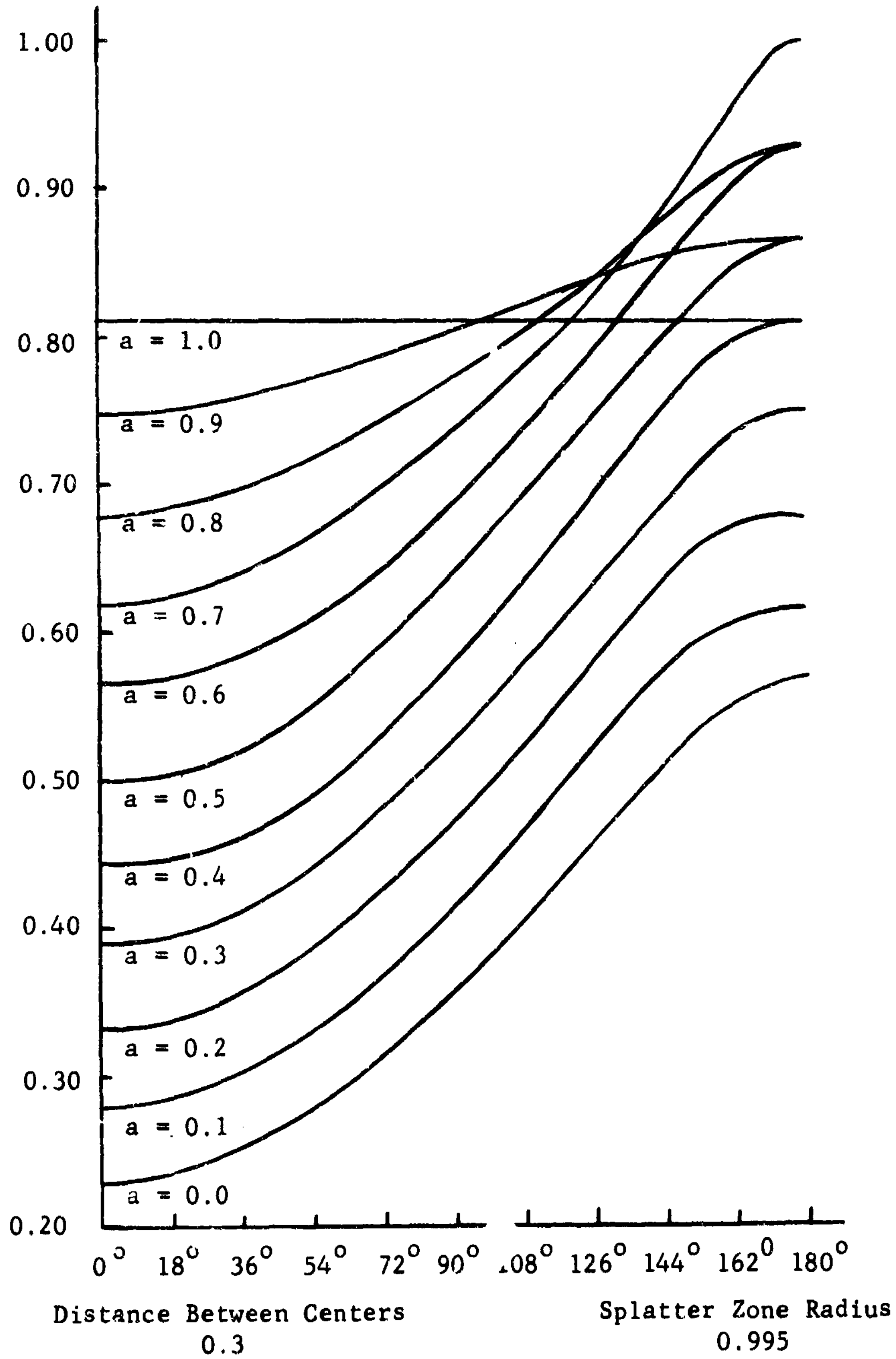
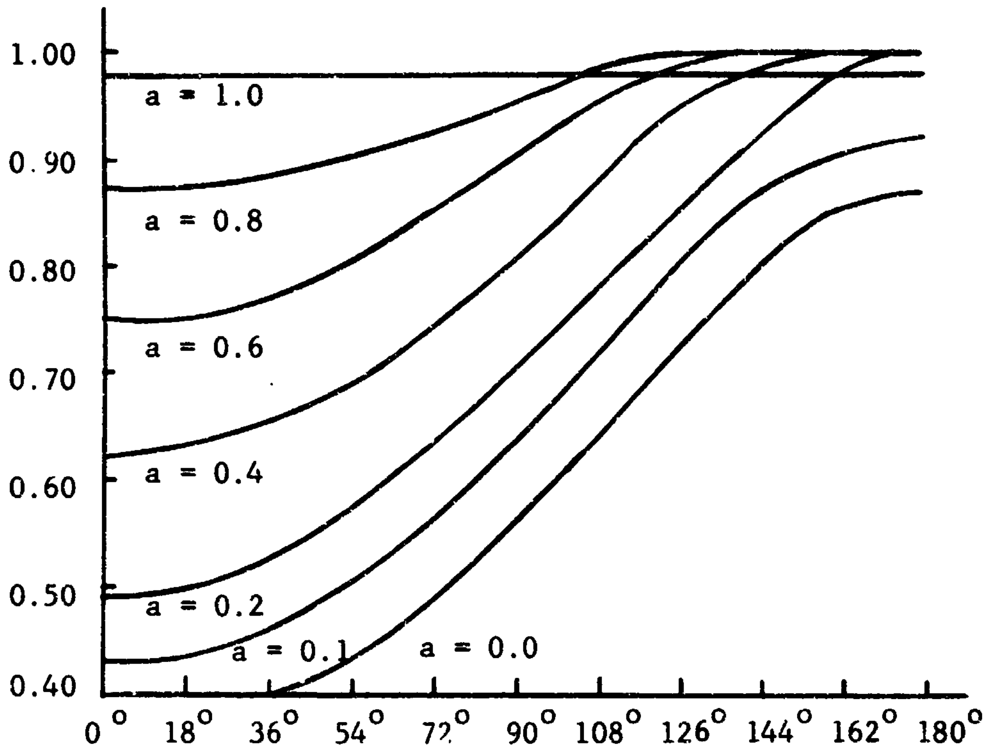


Figure 13

High Intensity Zone Radius 0.4

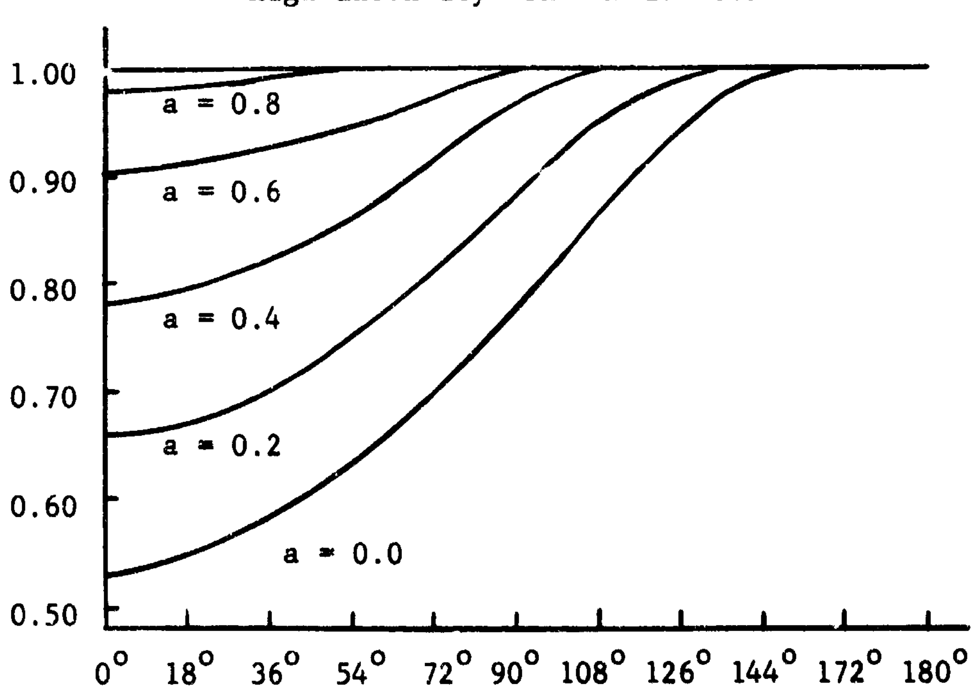


Distance Between Centers 0.4

Splatter Zone Radius 1.327

Figure 14

High Intensity Zone Radius 0.5



Distance Between Centers 0.5

Splatter Zone Radius 1.659

Figure 15

TABLE I

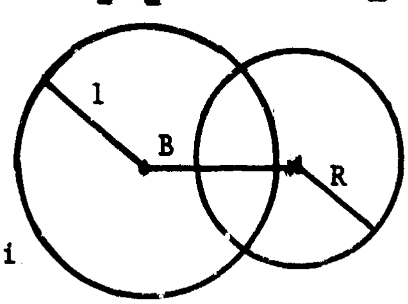
Area of Overlap Between Two Circles

A(R,B)

B \ R	0.1	0.2	0.3	0.4	0.5
0.5	0.01	0.04	0.09	0.16	0.2500
0.6	0.01	0.04	0.09	0.1600	0.233
0.7	0.01	0.04	0.0900	0.1466	0.206
0.8	0.01	0.0400	0.0788	0.[illegible]29	0.175
0.9	0.01000	0.0316	0.0612	0.0983	0.143
0.95	0.00802				
1.0	0.00490	0.0192	0.0421	0.0731	0.112
1.05	0.00183				
1.1	0.00000	0.0073	0.0240	0.0491	0.082
1.2	0.00000	0.0000	0.0088	0.0275	0.054
1.3	0.00000	0.0000	0.0000	0.0099	0.030
1.4	0.00000	0.0000	0.0000	0.0000	0.011
1.5	0.00000	0.0000	0.0000	0.0000	0.0000

$$\pi A(R,B) = \cos^{-1}\frac{B^2 + (1 - R^2)}{2B} + R^2 \cos^{-1}\frac{B^2 - (1 - R^2)}{2BR}$$

$$- \frac{1}{2}\sqrt{\left[(1 + R^2) - B^2\right]\left[B^2 - (R - 1)^2\right]}$$



R = ratio of circle radii

B = relative distances between centers of circles

TABLE II

Bomb Configuration (See Figure 3)

Radius of High Intensity Zone	Distance Between Center of High Intensity and Splatter Zone	Radius of Splatter Zone
0.1	0.124	0.224
0.2	0.247	0.447
0.3	0.371	0.671
0.4	0.494	0.894
0.5	0.618	1.118

TABLE III

Flame Thrower Configuration (See Figure 4)

Radius of High Intensity Zone	Distance Between Center of High Intensity and Splatter Zone	Radius of Splatter Zone
0.1	0.1	0.332
0.2	0.2	0.663
0.3	0.3	0.995
0.4	0.4	1.327
0.5	0.5	1.659

Table IV

Distance to Center of Splatter Zone

	0^{o}	18^{o}	36^{o}	54^{o}	72^{o}	90^{o}	108^{o}	126^{o}	144^{o}	162^{o}	180^{o}
a = 0.0											
r = 0.1	1.10	1.10	1.08	1.06	1.03	1.00	0.97	0.94	0.92	0.91	0.90
0.3	1.30	1.29	1.26	1.20	1.13	1.04	0.95	0.86	0.78	0.72	0.70
0.5	1.50	1.48	1.43	1.36	1.25	1.12	0.97	0.81	0.66	0.55	0.50
a = 0.1											
r = 0.1	1.00	1.00	0.98	0.96	0.94	0.91	0.87	0.85	0.82	0.81	0.80
0.3	1.20	1.19	1.16	1.10	1.03	0.95	0.86	0.76	0.68	0.62	0.60
0.5	1.40	1.38	1.34	1.26	1.16	1.03	0.88	0.73	0.58	0.45	0.40
a = 0.2											
r = 0.1	0.90	0.90	0.88	0.86	0.84	0.81	0.77	0.75	0.72	0.71	0.70
0.3	1.10	1.09	1.06	1.01	0.94	0.85	0.76	0.67	0.58	0.52	0.50
0.5	1.30	1.28	1.24	1.17	1.07	0.94	0.80	0.65	0.49	0.36	0.30
a = 0.3											
r = 0.1	0.80	0.80	0.78	0.76	0.74	0.71	0.68	0.65	0.62	0.61	0.60
0.3	1.00	0.99	0.96	0.91	0.84	0.76	0.67	0.58	0.49	0.42	0.40
0.5	1.20	1.19	1.14	1.07	0.98	0.86	0.72	0.57	0.42	0.27	0.20
a = 0.4											
r = 0.1	0.70	0.70	0.68	0.66	0.64	0.61	0.58	0.55	0.52	0.51	0.50
0.3	0.90	0.89	0.86	0.81	0.75	0.67	0.58	0.49	0.40	0.33	0.30
0.5	1.10	1.09	1.05	0.98	0.89	0.78	0.65	0.51	0.35	0.20	0.10

TABLE IV (Continued)

		0°	18°	36°	54°	72°	90°	108°	126°	144°	162°	180°
a = 0.5												
r = 0.1		0.60	0.60	0.58	0.56	0.54	0.51	0.48	0.45	0.42	0.41	0.40
0.3		0.80	0.79	0.76	0.72	0.66	0.58	0.50	0.40	0.31	0.23	0.20
0.5		1.00	0.99	0.95	0.89	0.81	0.71	0.59	0.45	0.31	0.16	0.00
a = 0.6												
r = 0.1		0.50	0.50	0.48	0.47	0.44	0.41	0.38	0.35	0.32	0.31	0.30
0.3		0.70	0.69	0.67	0.63	0.60	0.50	0.42	0.33	0.24	0.15	0.10
0.5		0.90	0.89	0.86	0.80	0.73	0.64	0.54	0.42	0.29	0.17	0.10
a = 0.7												
r = 0.1		0.40	0.40	0.39	0.38	0.34	0.32	0.29	0.26	0.23	0.21	0.20
0.3		0.60	0.59	0.57	0.53	0.49	0.42	0.35	0.27	0.19	0.09	0.00
0.5		0.80	0.79	0.76	0.72	0.66	0.58	0.50	0.40	0.31	0.23	0.20
a = 0.8												
r = 0.1		0.30	0.30	0.29	0.27	0.25	0.22	0.19	0.16	0.13	0.11	0.10
0.3		0.50	0.49	0.48	0.45	0.41	0.36	0.30	0.24	0.18	0.13	0.10
0.5		0.70	0.69	0.67	0.64	0.59	0.54	0.48	0.42	0.36	0.32	0.30
a = 0.9												
r = 0.1		0.20	0.20	0.19	0.18	0.16	0.14	0.12	0.09	0.06	0.03	0.00
0.3		0.40	0.40	0.39	0.38	0.34	0.32	0.29	0.25	0.23	0.21	0.20
0.5		0.60	0.60	0.58	0.56	0.54	0.51	0.48	0.45	0.42	0.41	0.40

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